Published by Institute of Physics Publishing for SISSA

Received: *January 23, 2007*
Revised: *April 7, 2007*
Accepted: *May 11, 2007*
Published: *May 16, 2007*

Meta-stable brane configuration with orientifold 6 plane

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Abstract: We present the intersecting brane configuration of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua in four dimensional $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(N_c)$ gauge theory with a symmetric flavor, a conjugate symmetric flavor and fundamental flavors. By studying the previously known supersymmetric M5-brane curve, the M-theory lift for this type IIA brane configuration, which consists of NS5-branes, D4-branes, D6-branes and an orientifold 6-plane, is analyzed.

Keywords: Duality in Gauge Field Theories, Brane Dynamics in Gauge Theories, D-branes, M-Theory.

Contents

1. Introduction

The dynamical supersymmetry breaking [1] occurs in $\mathcal{N} = 1$ SQCD with massive fundamental flavor where the masses are much smaller than the dynamical scale of the gauge sector. The supersymmetry is broken by the rank condition: not all the F-term equations can be satisfied. Contrary to the electric theory which has a vanishing superpotential, the dual magnetic theory has a superpotential term interacting between the magnetic mesons and the magnetic quarks. See, for example, [2]. In this construction of ISS model [1], both the mass for the flavors and the Seiberg duality play an important role. By deforming the electric theory through the mass term for quarks, one can add a superpotential. This can be interpreted as a linear term in magnetic mesons in the magnetic theory. Then, the F-term conditions in the dual magnetic theory can be obtained from these two contributions, linear term in magnetic mesons plus the Yukawa coupling term above. It turns out that the F-terms of the mesons cannot all vanish due to the fact that the ranks are different. On the other hand, the definition of meta-stable states is meaningful only when they are long-lived parametrically. For the particular range of the number of flavors where the Seiberg dual should be applied, one can make the meta-stable vacuum long-lived arbitrarily. Therefore, it is necessary to consider both the mass for the flavors and Seiberg dual magnetic theory in order to find out new meta-stable supersymmetry breaking vacua.

In the type IIA brane configuration, the quark masses correspond to the relative displacement of the D6-branes (0123789) and D4-branes (01236) along the 45 directions geometrically. The eigenvalues of mass matrix correspond to the locations of D6-branes in 45 directions. See, for example, [2]. On the other hand, the Seiberg duality in the classical brane picture can be achieved by exchanging the locations of the NS5-brane (012345) and

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NS5'-brane (012389) along x^6 direction each other [2]. The geometric misalignment of D4-branes connecting both NS5'-brane and D6-branes in the magnetic brane configuration can be interpreted as a nontrivial F-term conditions in the gauge theory side [3−6]. Then the F-term can be partially cancelled by both recombination of flavor D4-branes (connecting to NS5'-brane) with the color D4-branes (connecting to NS5-brane) and then movement of those into the 45 directions. This phenomenon in brane configuration corresponds to the fact that some entries in the dual quarks acquire nonzero vacuum expectation values to minimize the F-term in the gauge theory side. Since there are no D4-branes between NS5-brane and NS5'-brane, the magnetic gauge group is completely Higgsed. Moreover, the remaining flavor D4-branes connecting to NS5'-brane can move along 89 directions freely and independently since D6-branes and NS5'-brane are parallel and this geometric freedom corresponds to the classical pseudomoduli space of nonsupersymmetric vacua of the gauge theory.

In this paper, we study $\mathcal{N} = 1$ SU(N_c) gauge theory with a symmetric flavor S, a conjugate symmetric flavor $\widetilde{S}$ and N_f fundamental flavors Q and $\widetilde{Q}$[1] in the context of dynamical supersymmetric breaking vacua. The corresponding supersymmetric brane configuration in type IIA string theory was given by Landsteiner, Lopez and Lowe in [7] sometime ago. In the gauge theory analysis [8] alone, the turning on the superpotential $W = \mathrm{tr}(S\widetilde{S})^2$ was necessary to truncate the chiral ring and the dual description was given. From the brane analysis [7] where, in general, two other quartic terms in the superpotential are present, this quartic superpotential can be obtained from the corresponding $\mathcal{N} = 2$ theory by integrating the massive adjoint field out [2].

The point is that for the limit of infinite mass for the adjoint field, the above quartic superpotential vanishes, because the coefficient function appears as an inverse of mass, as in the electric theory of $\mathcal{N} = 1$ SQCD with flavors we mentioned before. Now we can add the mass term for quarks in the fundamental representation of the gauge group while keeping the infinite mass limit. Then we turn to the dual magnetic gauge theories by two standard brane motions [2] with appropriate linking number counting. It turns out that the dual magnetic theory giving rise to the meta-stable vacua is described by $\mathcal{N} = 1$ SU($2N_f - N_c$) gauge theory with dual matter contents and the superpotential consists of linear term in magnetic meson plus the coupling terms between the magnetic meson, dual quarks, dual symmetric tensor flavor, and dual conjugate symmetric tensor flavor (4.3). Some of the F-term conditions cannot be satisfied by rank condition. The final nonsupersymmetric minimal energy brane configuration for this theory is shown in figure 3.

In section 2, we review the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ SU(N_c) gauge theory with above matter contents. In section 3, we construct the Seiberg dual magnetic theory which is $\mathcal{N} = 1$ SU($3N_f - N_c + 4$) gauge theory with corresponding dual matters as well as various gauge singlets, by brane motion and linking number counting. In section 4, we consider the infinite mass limit for the adjoint field of $\mathcal{N} = 2$ theory in order to obtain nonsupersymmetric meta-stable minimum

[1] The type IIA brane configuration describing the nonsupersymmetric meta-stable minimum in this theory is proposed in [4].

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where the gauge group is given by $SU(2N_f - N_c)$ with matter contents and present the corresponding intersecting brane configuration of type IIA string theory. In section 5, we describe M-theory lift of the supersymmetry breaking type IIA brane configuration we have considered in section 4, by following the work of [5, 9]. Finally, in section 6, we give the summary of this paper and make some comments for the future directions.

For the relevant works on the meta-stable vacua in various and different contexts, we refer to some partial lists appeared in [10]–[29].

2. The $\mathcal{N} = 1$ supersymmetric electric brane configuration

The Seiberg-Witten curve of $\mathcal{N} = 2$ $SU(N_c)$ gauge theory with a symmetric flavor S, a conjugate symmetric flavor $\widetilde{S}$ and N_f fundamental flavors $Q, \widetilde{Q}$ was found in [7].[2] The corresponding type IIA brane configuration [30] consists of three NS5-branes (012345) which have different x^6 values, N_c D4-branes (01236) suspended between them, $2N_f$ D6-branes (0123789) and an orientifold 6 plane (0123789) of positive Ramond charge where a middle NS5-brane is located.[3] Let us introduce two complex coordinates

$$v \equiv x^4 + ix^5, \qquad w \equiv x^8 + ix^9.$$

We'll introduce these more precise way later when we consider 11-dimensional theory. According to $\mathbf{Z}_2$ symmetry of orientifold 6-plane(O6-plane)[4] sitting at $v = 0$ and $x^6 = 0$, the coordinates (x^4, x^5, x^6) transform as $-(x^4, x^5, x^6)$. The left NS5-brane with positive (negative) v is mirror to the right NS5-brane with negative (positive) v and the middle NS5-brane with positive v is mirror to the middle NS5-brane with negative v. Every D4-brane and D6-brane which do not pass through $v = 0$ should also have its mirror image, according to this rule. The distance along v direction of D4 branes is related to the mass of two index tensor matter $(S\widetilde{S})$ and we put all the D4-branes at $v = 0$. Of course, the D4-branes with positive x^6 are mirror to those with negative x^6. Then the distance along v direction of D6-branes provides the mass for the fundamental matter $(Q\widetilde{Q})$. For the equal masses, the v coordinate of N_f D6-branes with nonzero x^6 has only one fixed value and its N_f mirrors appear with opposite values of v and x^6. See figure 1 for clear view on the brane configuration.

By rotating the left and right NS5-branes from v direction toward $\pm w$ direction (equivalently introducing the mass for adjoint field of $\mathcal{N} = 2$ theory and integrating out this massive adjoint field [2]) respectively, one obtains $\mathcal{N} = 1$ theory. Among three NS5-branes,

[2] For an asymptotic free region [7], we should have $N_c > N_f + 2$.

[3] The orientifold action identifies the two factors of gauge group $SU(N_c) \times SU(N_c)$ and projects the bifundamental representation onto the symmetric representation [30, 7]. The brane configuration of $\mathcal{N} = 1$ supersymmetric gauge theory with $SU(N_L) \times SU(N_R)$ and matter in the bifundamental and fundamental representations was found in [31, 32]. See also the relevant work in [33].

[4] For the negative Ramond charge of orientifold 6-plane with same brane configuration [30], the matter contents in the gauge theory side are changed into an antisymmetric flavor A and a conjugate antisymmetric flavor $\widetilde{A}$ as well as fundamentals Q and $\widetilde{Q}$. We'll concentrate on the case with positive O6-plane and the other case with negative O6-plane will be similar to what we present here and can be done without any difficulty by recognizing the dependence on the negative charge in various places.

the middle NS5-brane is stuck on an O6-plane as in figure 1 and the outer two NS5-branes can be rotated in a $\mathbf{Z}_2$ symmetric manner due to the presence of orientifold 6 plane. That is, if the left NS5-brane rotates by an angle θ in (v, w) plane, denoted by $NS5_\theta$-brane [2], the mirror image of this NS5-brane, the right NS5-brane, is rotated by an angle $-\theta$ in the same plane, denoted by $NS5_{-\theta}$-brane. For more details, see the figure 1. At the moment, the angle θ is greater than 0 and less than $\pi/2$. We will come to the case for $\theta = \pi/2$ later section. We also rotate the N_f D6-branes and make them be parallel to $NS5_\theta$ and denote them as $D6_\theta$(the notation is not good because the angle between the unrotated D6-branes and $D6_\theta$-branes is equal to $\pi/2 - \theta$, not θ) and its mirrors N_f D6-branes appear as $D6_{-\theta}$. There is no coupling between the adjoint field and the quarks since the rotated $D6_\theta$-branes are parallel to the rotated $NS5_\theta$-brane [31, 2]. For this brane setup, the classical superpotential is given by a quartic term for tensor matter and mass term for quarks as follows[5]

$$W = \frac{1}{\mu} \operatorname{tr}(S\widetilde{S})^2 + \operatorname{tr} mQ\widetilde{Q}$$

where S and $\widetilde{S}$ represent a flavor of symmetric and a conjugate symmetric tensor representations of $\mathrm{SU}(N_c)$ and the adjoint mass μ is given by the rotation angle θ through [2]

$$\mu \equiv \tan\theta.$$

The field theory analysis [8][6] corresponding to this superpotential which was necessary to truncate the chiral ring (with massless quarks) provides the dual description. Note that there exists an extra global $\mathrm{U}(1)_S$ symmetry acting on S and $\widetilde{S}$ only and is not realized geometrically in the brane configuration [7]. The masses of quarks are given by the location of D6-branes in v direction (Note that all the D4-branes are located at $v = 0$) as we mentioned before. If we leave the D6-branes parallel to the O6-plane, there will be more terms in the superpotential [7].[7] After this rotation, the three NS5-branes intersect at a point in (4589) directions implying that the mass for symmetric flavor or conjugate symmetric flavor($S\widetilde{S}$) vanishes. For the infinite limit of μ, the superpotential becomes $\operatorname{tr} mQ\widetilde{Q}$ which is the same superpotential for the $\mathcal{N} = 1$ SQCD with massive flavors. For the more general brane configurations with nonzero $S\widetilde{S}$, we refer to [7, 32].

[5] If there exist k coincident $NS5_\theta$-branes (and their mirrors also), then the superpotential with massless quarks takes the form $W = \operatorname{tr}(S\widetilde{S})^{k+1}$ and the corresponding field theory analysis on the magnetic dual was given in [8]. One can understand the quartic superpotential by writing the full superpotential as $W = \mu\Phi^2 + S\Phi\widetilde{S}$ and integrating the adjoint field Φ out.

[6] When the baryonic operator $B_n = S^n Q^{N_c - n} Q^{N_c - n}$ gets an expectation value, then the initial gauge group $\mathrm{SU}(N_c)$ is broken to $\mathrm{SO}(n)$ with a symmetric tensor $\widetilde{S}$ and $2N_f$ vectors and the superpotential will be $\operatorname{tr} \widetilde{S}^2$ [8]. Similarly, when the baryonic operator $B_n = A^n Q^{N_c - 2n}$ gets an expectation value with negative O6-plane charge, then the initial gauge group $\mathrm{SU}(N_c)$ is broken to $\mathrm{Sp}(n)$ with a conjugate antisymmetric tensor $\widetilde{A}$ and N_f flavors and the superpotential will be $\operatorname{tr} \widetilde{A}^2$.

[7] According to the result of [34], the dual magnetic gauge group from brane configuration is given by $\mathrm{SU}(2N_f - N_c + 4)$ and the superpotential has more terms with the same matter contents as those in electric theory. Of course, these extra two terms in the superpotential can be interpreted as mesonic perturbation in the magnetic theory [34].

Now one summarizes the supersymmetric electric brane configuration with their worldvolumes in type IIA string theory as follows.

- Middle NS5-brane with worldvolume (012345) with $w = 0 = x^6$.
- Left $NS5_\theta$-brane with worldvolume by both (0123) and two spatial dimensions in (v, w) plane and with negative x^6.
- Right $NS5_{-\theta}$-brane with worldvolume by both (0123) and two spatial dimensions in (v, w) plane and with positive x^6.
- Left N_f $D6_\theta$-branes with worldvolume by both (01237) and two spatial dimensions in (v, w) plane and with negative x^6. Before the rotation, the distance from D4-branes in the v direction is nonzero.
- Right N_f $D6_{-\theta}$-branes with worldvolume by both (01237) and two space dimensions in (v, w) plane and with positive x^6.
- O6-plane with worldvolume (0123789) with $v = 0 = x^6$.
- N_c D4-branes with worldvolume (01236) with $v = 0 = w$.

We draw the type IIA electric brane configuration in figure 1 which was basically given in [7, 2] already but the only difference is to put D6-branes in the nonzero v direction in order to obtain nonzero masses for the quarks which are necessary to obtain the meta-stable vacua as we observed in the previous section.[8] Also one can imagine the brane configuration for $\theta = \pi/2$ from figure 1.

3. The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration

As explained in the introduction, the Seiberg duality is crucial for the construction of meta-stable vacua. In order to obtain the magnetic dual theory in the context of brane configuration, we move the D6 and NS5-branes through each other [2] and use the linking numbers [36] for the computation of D4-branes which are created during this process. In other words, the magnetic theory in the brane picture is obtained by interchanging D6-branes and NS5-branes with their mirror images while preserving the linking number. The linking number [36] for the D6-branes is given by[9]

$$L_6 = \frac{1}{2}(n_{5L} - n_{5R}) + n_{4R} - n_{4L} \tag{3.1}$$

[8] One of the variants of this brane configuration (no middle NS5-brane) provides $\mathcal{N} = 1$ symplectic or orthogonal gauge groups with N_f flavors depending on the charges of O6-plane with a quartic superpotential (after integrating out the massive adjoint field) for the quarks: Left $NS5_\theta$-brane, Right $NS5_{-\theta}$-brane, $2N_f$ D6-branes which are not parallel to $NS5_{\pm\theta}$-branes, O6-plane and N_c D4-branes [34, 35, 2]. It is not clear how the meta-stable brane construction from the magnetic dual description of this theory with infinite mass for the adjoint field arises. Without D6-branes, this theory has a superpotential $W = \frac{1}{\mu}\,\mathrm{tr}(S\widetilde{S})^2$, as usual.

[9] In [2], the linking number has opposite sign but the overall sign is not important when we use the conservation of linking number between before the brane motion and after the brane motion.

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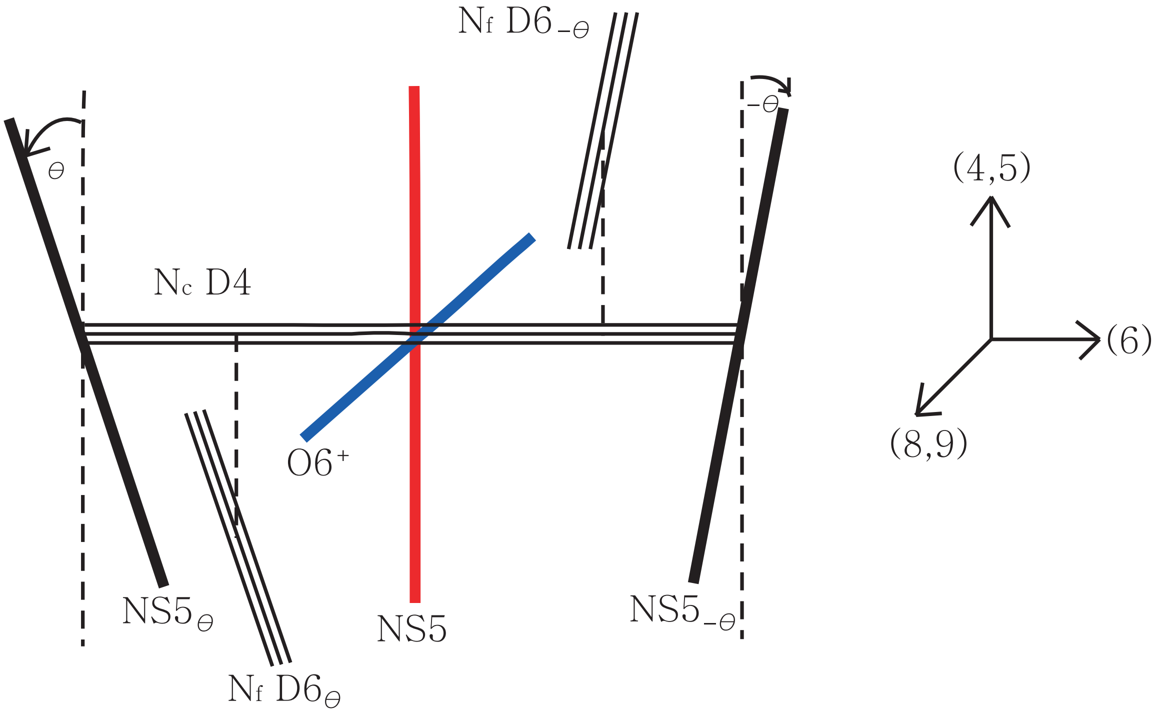

Figure 1: The $\mathcal{N}=1$ supersymmetric electric brane configuration for the SU(N_c) gauge theory with a symmetric flavor S, a conjugate symmetric flavor $\widetilde{S}$ (which are massless) that are strings stretching between D4-branes located at the left hand side of O6-plane and those at the right hand side of O6-plane and N_f fundamental massive flavors $Q, \widetilde{Q}$ that are strings stretching between D6-branes and D4-branes. The origin of the coordinates (x^6, v, w) is located at the intersection of NS5-brane and O6-plane.

where $n_{5L,R}$ are the NS5-branes to the left or right of the D6-branes and $n_{4L,R}$ are the D4-branes to the left or right of the D6-branes. After we move the left $D6_\theta$-branes to the right all the way(and their mirrors, right $D6_{-\theta}$-branes to the left) past the NS5-brane and $NS5_{-\theta}$-brane, the linking number L_6 of a $D6_\theta$-brane becomes $L_6 = 1 - n_{4L}$, because $n_{5L} = 2$ and $n_{5R} = 0 = n_{4R}$, which should be equal to the original $L_6 = -1$ because $n_{5L} = 0 = n_{4L} = n_{4R}$ and $n_{5R} = 2$ before the brane motion, according to the conservation of linking number. Then the n_{4L} becomes 2 and we must add $2N_f$ D4-branes to the left side of all N_f $D6_\theta$-branes (and their mirrors). See figure 2 for the creation of these D4-branes.

Next, we move the left $NS5_\theta$-brane to the right all the way past O6-plane (and its mirror, right $NS5_{-\theta}$-brane to the left), the linking number of $NS5_\theta$-brane, $L_5 = \frac{N_f}{2} + \frac{1}{2}(4) - N + 2N_f$ using the general formula[10]

$$L_5 = \frac{1}{2}(n_{6L} - n_{6R}) + n_{4R} - n_{4L} \tag{3.2}$$

where $n_{6L,R}$ are the D6-branes to the left or right of the NS5-branes, $n_{4L,R}$ are the D4-branes to the left or right of the NS5-branes and the linking number for O6-plane is given

[10] One can also add the contribution from O6-plane explicitly as in [37]. In our convention, $n_{6L,R}$ includes the linking number from O6-plane.

by 4 D6-branes. Originally it was $L_5 = -\frac{N_f}{2} - \frac{1}{2}(4) + N_c$ from figure 1 before the brane motion. This leads to the fact that the number of D4-branes along the x^6 direction becomes

$$N = 3N_f - N_c + 4 \tag{3.3}$$

for the dual magnetic gauge group.[11] This result[12] agrees with the field theory analysis in [8]. It is easy to check that the linking numbers for the middle NS5-brane are consistent with this dual configuration. The dual brane configuration[13] is shown in figure 2. In particular, $2N_f$ D4-branes connecting between $NS5_{-\theta}$-brane and $D6_{-\theta}$-branes are the singlets corresponding to the mesons

$$M_0 \equiv Q\widetilde{Q} \qquad \text{and} \qquad M_1 \equiv Q\widetilde{S}S\widetilde{Q}. \tag{3.4}$$

The former occurs when $D6_\theta$-branes cross $NS5_{-\theta}$-brane while the latter occurs when $D6_\theta$-branes cross a middle NS5-brane. They are gauge singlets under SU(N) and are fundamentals $(\mathbf{N_f}, \mathbf{N_f})$ under the global flavor symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$. From the $\mathcal{N} = 1$ electric/magnetic duality we expect to get the dual gauge group $\mathrm{SU}(3N_f - N_c + 4)$ with a symmetric flavor s, a conjugate symmetric flavor $\widetilde{s}$ and N_f fundamental flavors $q, \widetilde{q}$ as well as various gauge singlets. By adding a linear term in M_0, the superpotential term [8] becomes

$$W_{\text{dual}} = (s\widetilde{s})^2 + M_0 q\widetilde{s}s\widetilde{q} + M_1 q\widetilde{q} + P_0 q\widetilde{s}q + \widetilde{P}_0\widetilde{q}s\widetilde{q} + mM_0 \tag{3.5}$$

where two other singlets are given by $P_0 \equiv Q\widetilde{S}Q$ and $\widetilde{P}_0 \equiv \widetilde{Q}S\widetilde{Q}$ which are symmetric in their flavor indices.[14] This theory has the $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_S \times \mathrm{U}(1)_B \times \mathrm{U}(1)_R$ global flavor symmetry. The explicit matter field transformations under these symmetries are given in [8]. In the dual theory,[15] the operator B_n defined in footnote 6 is mapped to $b_n = s^{(2N_f+4)-n} q^{N_f+n-N_c} q^{N_f+n-N_c}$ and when this operator gets an expectation value, the dual SU(N) gauge theory is broken to $\mathrm{SO}(\widetilde{n} = 2N_f - n + 4)$ and the low energy has $2N_f$ matter fields and a symmetric tensor [8]. If we turn on the operator $\widetilde{B}_n = \widetilde{S}^n \widetilde{Q}^{N_c - n} \widetilde{Q}^{N_c - n}$ getting the expectation values further in the electric theory, then the initial gauge group is broken to SO(n) with $2N_f$ vectors and the dual description on this operator can be done similarly.

[11] For magnetic IR free region, we should have $N_f > N - 2$ and this implies that $\frac{N_c}{3} - \frac{4}{3} < N_f < \frac{N_c}{2} - 1$.

[12] It is natural to ask whether this analysis can be generalized to the arbitrary superpotential given in the footnote 5. It is straightforward to take the Seiberg dual successively and it turns out one arrives at the dual magnetic gauge group $\mathrm{SU}((2k+1)N_f - N_c + 4k)$ where $2kN_f$ term comes from k coincident $NS5_{\pm\theta}$-branes, N_f term comes from a middle NS5-brane and the presence of $4k$ in the dual gauge group is due to the O6-plane [2].

[13] The linking number counting without O6-plane was given in [31] and similar counting without a middle NS5-brane was given in [34]. Also other example of linking number counting appears in [38].

[14] When D6-branes are unrotated [34] and not parallel to rotated NS5-branes, then in the superpotential there exist extra terms $(Q\widetilde{Q})^2 + Q\widetilde{S}S\widetilde{Q}$ in the electric theory and these will appear as $M_0^2 + M_1$ in the magnetic theory from the point of brane configuration, as pointed out in [34].

[15] It was found in [8] that with the deformation for the mass of N_f-th quark, the vacuum giving the expectation values $< q_{N_f} >, < \widetilde{q}_{N_f} >, < s >$ and $< \widetilde{s} >$ breaks the magnetic gauge group $\mathrm{SU}(3N_f - N_c + 4)$ into $\mathrm{SU}(3N_f - N_c + 1)$ with $(N_f - 1)$ remaining light flavors. The M_0 and M_1 equations of motion imply this vacuum of the theory. Of course, for the more general superpotential (more than quartic superpotential), this kind of deformation is applicable.

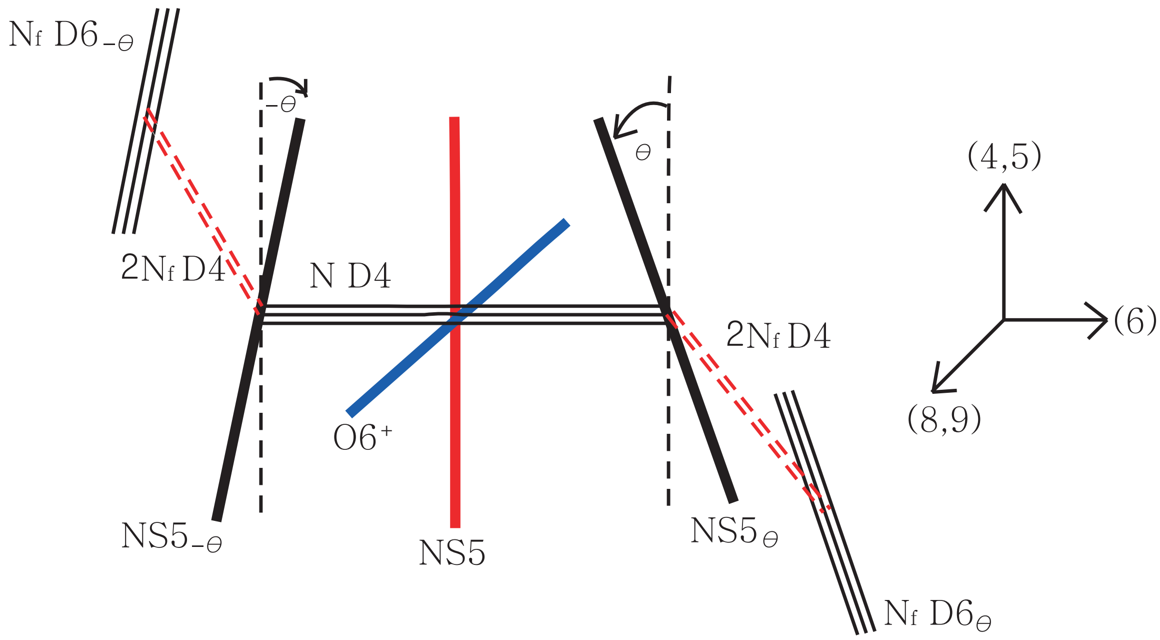

Figure 2: The $\mathcal{N}=1$ supersymmetric magnetic brane configuration for the SU$(N=3N_f-N_c+4)$ gauge theory with a symmetric flavor s, a conjugate symmetric flavor $\widetilde{s}$ and N_f fundamental flavors $q,\widetilde{q}$. The $2N_f$ D4-branes connecting between $NS5_{-\theta}$-brane and $D6_{-\theta}$-branes are the dual gauge singlets corresponding to the mesons M_0 and M_1 (3.4). As will be explained in section 4, in the $\mu\to\infty$ limit (or $\theta\to\pi/2$ limit), only N_f D4-branes connecting NS5'-brane and D6-branes appear (and their mirrors). In the superpotential, it contains either M_0 or M_1-dependent term depending on which way the brane motion occurs.

4. Nonsupersymmetric meta-stable brane configuration

We have seen that there are $2N_f$ D4-branes connecting between $NS5_{-\theta}$-brane and $D6_{-\theta}$-branes (and their mirrors) for finite μ in figure 2. In order to describe $\mu=\infty$ limit case in this section, one needs to see each step from the electric theory to magnetic theory more closely. For fixed left and right NS5'-branes, the D6-branes can be located either at the parallel position with NS5'-branes or at the nonparallel position with them in the electric theory. We are interested in the case where they are perpendicular or parallel to each other because only these two cases provide the situation where the above $2N_f$ D4-branes creation can be decomposed into half N_f D4-branes and half N_f D4-branes independently. Let us describe two different cases separately as follows.

1. *When the D6-branes are perpendicular to a middle NS5-brane in the electric theory*

   So far, the rotation angle is less than $\pi/2$. In this section, we consider the case of $\theta=\pi/2$. Then the left $NS5_\theta$-brane in the electric theory is parallel to the right $NS5_{-\theta}$-brane and both $D6_{\pm\theta}$-branes become unrotated D6-branes which are parallel to the above $NS5'\equiv NS5_{\pm\pi/2}$ brane. During the process of previous section when $\theta=\pi/2$, the creation of D4-branes is a little bit changed because when a D6-brane crosses over NS5'-brane and when NS5'-brane crosses O6-plane or D6-brane, there is no creation of D4-branes. In other words, a D4-brane is created whenever a NS5-brane crosses a D6-brane as long as the NS5-brane and a D6-brane are not parallel.

After we move the left D6-branes to the right all the way (and their mirrors, right D6-branes to the left) past the middle NS5-brane and the right NS5'-brane, the linking number L_6 of a single D6-brane from (3.1) becomes $L_6 = \frac{1}{2} - n_{4L}$, since $n_{5L} = 1$ from a middle NS5-brane, which should be equal to the original linking number $L_6 = -\frac{1}{2}$ because $n_{5R} = 1$ from a middle NS5-brane. Then we should add N_f D4-branes, corresponding to the meson M_1, to the left side of all the right N_f D6-branes (and their mirrors) in figure 2. Note that the difference, compared with previous case where $0 < \theta < \pi/2$, appears here when a D6-brane cross the right NS5'-brane. Due to the parallelness of these, there is no creation of D4-branes.

Next, we move the left NS5'-brane to the right all the way past O6-plane (and its mirror, right NS5'-brane to the left), and then the linking number of NS5'-brane from (3.2), $L_5 = -N + N_f$ from $n_{4L} = N$ and $n_{4R} = N_f$. Originally, it was $L_5 = N_c$ because $n_{4R} = N_c$ in figure 1. This implies that N becomes

$$N = N_f - N_c. \tag{4.1}$$

There is no D4-brane creation when NS5'-brane crosses an O6-plane because they are parallel to each other. This is a new feature, compared with the case of finite μ.

Finally, we are left with figure 2 with $\theta = \pi/2$ except that the number of D4-branes connecting between D6-branes and NS5'-brane is N_f not $2N_f$ and the gauge group is $\mathrm{SU}(N = N_f - N_c)$. These N_f D4-branes correspond to the singlet M_1 (3.4). In this case, the superpotential is given by $W_{\text{dual}} = \mathrm{tr}\, M_1 q\tilde{q} + \mathrm{tr}\, m M_0$ where the first term comes from the superpotential with M_1 for finite μ case in previous section (3.5) and the second term comes from the mass term for the quarks in the context of magnetic theory. This magnetic theory doesn't produce the meta-stable vacua because all the F-term equations are satisfied.

2. *When the D6-branes are parallel to a middle NS5-brane or perpendicular to NS5'-branes in the electric theory*

   Let us first rotate D6-branes to the v direction and make them to be parallel to a middle NS5-brane (and its mirrors also). Note that in this case, the superpotential with finite μ has more terms as we mentioned in the footnote 14. After we move the left D6-branes to the right all the way (and their mirrors, right D6-branes to the left) past the middle NS5-brane and the right NS5'-brane, the linking number L_6 of a single D6-brane (3.1) becomes $L_6 = \frac{1}{2} - n_{4L}$, because $n_{5L} = 1$ from the right NS5'-brane, which should be equal to the original linking number $L_6 = -\frac{1}{2}$ because $n_{5R} = 1$ from the left NS5'-brane. Then we should add N_f D4-branes, corresponding to the meson M_0, to the left side of all the right N_f D6-branes (and their mirrors) in figure 2. Note that the difference, compared with previous case where $0 < \theta < \pi/2$, appears here when a D6-brane crosses the middle NS5-brane. Due to the parallelness of these, there is no creation of D4-branes.

Next, we move the left NS5'-brane to the right all the way past O6-plane (and its mirror, right NS5'-brane to the left), and then the linking number of NS5'-brane in (3.2), $L_5 = \frac{N_f}{2} - N + N_f$ because $n_{4L} = N$, $n_{4R} = N_f$ and $n_{6L} = N_f$. Originally, it was $L_5 = -\frac{N_f}{2} + N_c$ from the fact that $n_{4R} = N_c$ and $n_{6R} = N_f$ in figure 1. This implies that N becomes

$$N = 2N_f - N_c. \tag{4.2}$$

The N_f-dependent term in finite μ case (3.3) is distributed as N_f in previous case (4.1) and $2N_f$ in present case (4.2). There is no D4-brane creation when NS5'-brane crosses an O6-plane because they are parallel to each other. Finally, we are left with figure 2 with $\theta = \pi/2$ except that the number of D4-branes connecting between D6-branes and NS5'-brane is N_f not $2N_f$ and the gauge group is $\mathrm{SU}(N = 2N_f - N_c)$ after we rotate D6-branes to the original positions and make them to be parallel to NS5-brane.[16] These N_f D4-branes correspond to the singlet M_0 (3.4). The left hand side of O6-plane including NS5-brane is exactly the same magnetic brane configuration describing the moduli space in massive $\mathcal{N} = 1$ SQCD, in particular, the figure 6 of [5]. The magnetic superpotential corresponding to the electric superpotential $W = m \,\mathrm{tr}\, Q\widetilde{Q}$ is given by

$$W_{\mathrm{dual}} = \mathrm{tr}\, M_0 q \widetilde{s} s \widetilde{q} + \mathrm{tr}\, m M_0 \tag{4.3}$$

where the first term comes from the superpotential with M_0 for finite μ case in previous section (3.5) and the second term comes from the mass term for the quarks in the context of magnetic theory. Here q and $\widetilde{q}$ are fundamental and antifundamental for the gauge group indices and antifundamentals for the flavor indices. The fields s and $\widetilde{s}$ are symmetric tensor and conjugate symmetric tensor for the gauge group indices respectively with no flavor indices. Then, $q\widetilde{s}s\widetilde{q}$ has rank N while m has a rank N_f. Therefore, the F-term condition, the derivative the superpotential W_{dual} with respect to M_0, cannot be satisfied if the rank N_f exceeds N. This is so-called rank condition given by [1].[17]

The classical moduli space of vacua can be obtained from F-term equations. From the F-terms F_q and $F_{\widetilde{s}}$, one gets $\widetilde{s}s\widetilde{q}M_0 = 0 = s\widetilde{q}M_0 q$. Similarly, one obtains $\widetilde{q}M_0 q\widetilde{s} = 0 = M_0 q \widetilde{s} s$ from the F-terms F_s and $F_{\widetilde{q}}$. Moreover, there is a relation $q\widetilde{s}s\widetilde{q} + m = 0$ from the F-term F_{M_0}. From the conditions $s\widetilde{q}M_0 = 0 = M_0 q\widetilde{s}$ which satisfy the above four equations for nonzero vacuum expectation values $q, \widetilde{q}, s$ and $\widetilde{s}$, one can fix the form of M_0 and part

[16] For an IR free magnetic region, we should have $N < N_f + 2$. With this condition and $N > 0$, the range for N_f is given by $\frac{N_c}{2} < N_f < N_c + 2$.

[17] Since the dual theory is in the IR free range, the Kahler potential is regular around the origin of the field space and can be expanded. Since the meson M_0 is identified with $Q\widetilde{Q}$ in the electric description, its dimension is not equal to 1. Therefore $1/\Lambda_1^2$ appears in the Kahler potential $M_0^\dagger M_0$ where Λ_1 is an electric scale. By redefining $M_0, q, \widetilde{q}, s$, and $\widetilde{s}$, their coefficients in the Kahler potential are normalized to be 1. Also the coupling connecting the scale Λ_1 and magnetic scale $\widetilde{\Lambda}_1$ and the mass matrix m are redefined appropriately. The higher order corrections in the Kahler potential are negligible.

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of $q\widetilde{s}$ and $s\widetilde{q}$ which can be fixed by using $F_{M_0} = 0$ further and one obtains the following solutions

$$< q\widetilde{s} >= \begin{pmatrix} \sqrt{m}e^{\phi}\mathbf{1}_N \\ 0 \end{pmatrix}, < s\widetilde{q} >= \begin{pmatrix} \sqrt{m}e^{-\phi}\mathbf{1}_N & 0 \end{pmatrix}, < M_0 >= \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \mathbf{1}_{N_f - N} \end{pmatrix} \quad (4.4)$$

where $\Phi_0 \mathbf{1}_{N_f-N}$ is an arbitrary $(N_f - N) \times (N_f - N)$ matrix and the zeros of $< q\widetilde{s} >$ and $< s\widetilde{q} >$ are $(N_f - N) \times N$ and $N \times (N_f - N)$ zero matrices respectively. Similarly, the zeros of $N_f \times N_f$ matrix M_0 are assumed also. Then Φ_0 and $(\sqrt{m}e^{\phi}, \sqrt{m}e^{-\phi})$ parametrize a pseudo-moduli space. Let us expand around on a point on (4.4) as done in [1]. That is,

$$q = \begin{pmatrix} q_0 \mathbf{1}_N + \frac{1}{\sqrt{2}}(\delta\chi_+ + \delta\chi_-)\mathbf{1}_N \\ \delta\varphi \end{pmatrix}, \qquad \widetilde{s} = \left(\widetilde{s}_0 + \delta\widetilde{X}\right) \mathbf{1}_{N\times N}, \quad s = (s_0 + \delta X)\, \mathbf{1}_{N\times N},$$

$$\widetilde{q} = \begin{pmatrix} \widetilde{q}_0 \mathbf{1}_N + \frac{1}{\sqrt{2}}(\delta\chi_+ - \delta\chi_-)\mathbf{1}_N & \delta\widetilde{\varphi} \end{pmatrix}, \quad M_0 = \begin{pmatrix} \delta Y & \delta Z \\ \delta\widetilde{Z} & \Phi_0 \mathbf{1}_{N_f-N} \end{pmatrix}.$$

Then the superpotential becomes

$$\begin{aligned} W_{\rm dual}^{\rm fluct} &= \Phi_0 \left(\delta\varphi\; \widetilde{s}_0\; s_0\; \delta\widetilde{\varphi} + m\right) + \delta Z\; \delta\varphi\; \widetilde{s}_0\; s_0\; \widetilde{q}_0 + \delta\widetilde{Z}\; q_0\; \widetilde{s}_0\; s_0\; \delta\widetilde{\varphi} \\ &\quad + \left(\frac{1}{\sqrt{2}}\delta Y\; \delta\chi_+\; \widetilde{s}_0\; s_0\; \widetilde{q}_0 + \cdots\right) + (\text{cubic}) \end{aligned} \quad (4.5)$$

where (cubic) stands for the terms that are cubic or higher in the fluctuations. Then to quadratic order, the model splits into two sectors where the first piece(the first line of (4.5)) is an O'Raifeartaigh type model and the second piece is supersymmetric and will not contribute to the supertrace(the second line of (4.5)). The fields $\delta\chi_\pm, \delta Y, \delta X$ and $\delta\widetilde{X}$ couple to the supersymmetry breaking fields $\delta\varphi$ and $\delta\widetilde{\varphi}$ via terms of cubic and higher order in the fluctuations. Then the remaining relevant terms of superpotential are given by

$$W_{\rm dual}^{\rm rel} = \Phi_0 \left(\delta\hat{\varphi}\; \delta\hat{\widetilde{\varphi}} + m\right) + \delta Z\; \delta\hat{\varphi}\; s_0\; \widetilde{q}_0 + \delta\widetilde{Z}\; q_0\; \widetilde{s}_0\; \delta\hat{\widetilde{\varphi}}$$

where $\delta\hat{\varphi} \equiv \delta\varphi\; \widetilde{s}_0$ and $\delta\hat{\widetilde{\varphi}} \equiv s_0\; \delta\widetilde{\varphi}$. At one loop, the effective potential $V_{\rm eff}^{(1)}$ for Φ_0 can be obtained from this relevant part of superpotential which consists of the matrix M and N(or $\hat{M}$ and $\hat{N}$) of [39]. One can compute these matrices for simplest rank case, as in the appendix A of [39]. His defining function $\mathcal{F}(v^2)$ can be computed and using the relation(the equation (2.14) of [39]) of $m^2_{\Phi_0}$ and $\mathcal{F}(v^2)$(and noting that the seconde piece of $m^2_{\Phi_0}$ will vanish), one gets that $m^2_{\Phi_0}$ will contain $\frac{m}{8\pi^2}(\log 4 - 1) > 0$, by taking the limit where $q_0\widetilde{s}_0 \rightarrow \sqrt{m}e^{\phi}$ and $s_0\widetilde{q}_0 \rightarrow \sqrt{m}e^{-\phi}$, as in (4.4). This implies these vacua are stable. In the brane configuration from figure 2, since the N D4-branes can slide along the two NS5'-branes when $\theta = \frac{\pi}{2}$, the vacuum expectation values of s and $\widetilde{s}$ can be diagonalized.

By recombination of N of D4-branes connecting D6-branes and NS5'-brane with those connecting NS5'-brane and NS5-brane and moving them in v direction (and their mirrors) from figure 2, the minimal energy supersymmetry breaking brane configuration is shown in

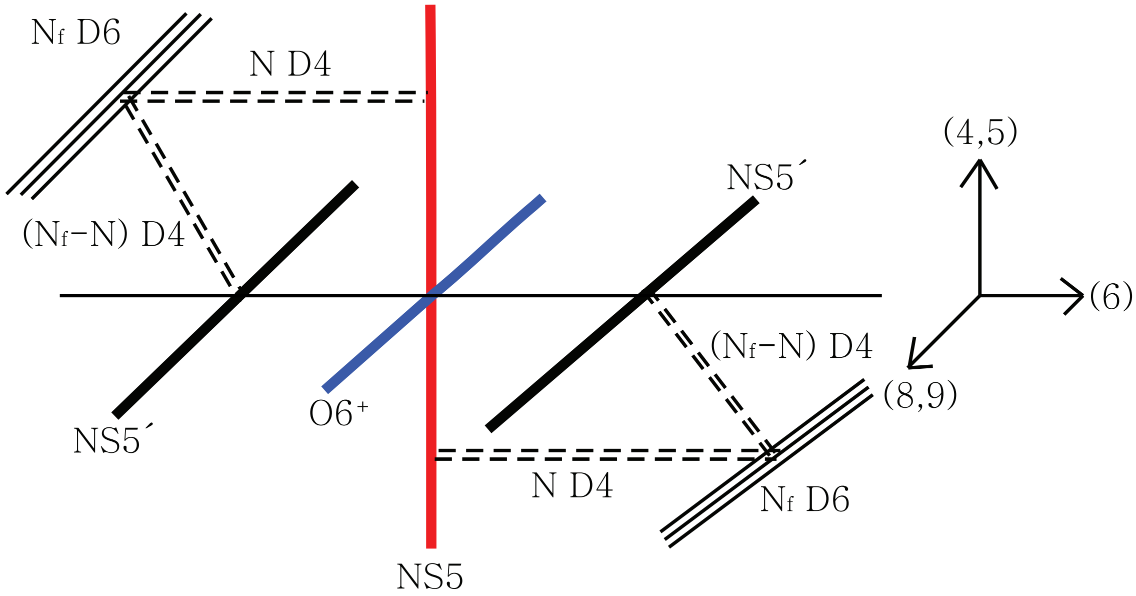

Figure 3: The nonsupersymmetric minimal energy brane configuration for the SU($N = 2N_f - N_c$) gauge theory with a symmetric flavor s, a conjugate symmetric flavor $\widetilde{s}$ and N_f fundamental flavors $q, \widetilde{q}$. The N_f D4-branes connecting between NS5'-brane and D6-branes are the dual gauge singlet corresponding to the meson M_0 (3.4). When we consider the M-theory lift of this brane configuration, we move the left NS5'-brane to the v direction holding everything else fixed, instead of moving D6-branes. The corresponding mirrors and D4-branes are placed appropriately during this process.

figure 3[18] that was observed also in [4]. Some entries in the dual quarks $q, \widetilde{q}$ and dual tensors $s, \widetilde{s}$ acquire the nonzero expectation values in terms of the eigenvalues of mass matrix m to minimize the F-term F_{M_0} in this dual gauge theory. When one gets the nonzero vacuum expectation values for $< M_0 >, < q >$, and $< \widetilde{q} >$, then the separation of the D4-branes along the middle NS5-brane corresponds to the mass of two index tensor ($s\widetilde{s}$). On the other hand, for the nonzero expectation values of $< M_0 >, < s >$ and $< \widetilde{s} >$, the separation of the D4-branes along the middle NS5-brane corresponds to the mass for the dual quarks ($q\widetilde{q}$).

5. M-theory description of nonsupersymmetric meta-stable brane configuration

The M5-brane spans (0123) directions and wraps on a Riemann surface inside (4568910) directions. The Taub-NUT space (45610) is parametrized by two complex variables (v, y) and flat two-dimensions (89) are by a complex variable w. The mass dimensions of these

[18] One can compute the energy of the nonsupersymmetric vacuum using either the effective field theory or the DBI action using the length of D4-branes, as done in [5]. In the limit of $|\Delta x| \ll L_0$ where Δx is a distance of D6-branes along the v direction and L_0 is a distance between D6-branes and NS5'-brane with massless flavors, the two expressions are the same. Moreover, the energy difference between the tachyonic state and the vacuum agrees with the same quantity from field theory in the limit $|\Delta x| \ll L_0$.

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variables are given by $[v] = 1, [y] = 2N_c, [w] = 2$ respectively [5]. The mass dimension for v can be seen from the corresponding Seiberg-Witten curve. For large v, since w goes to $\pm\mu v$ for the left and right $NS5_{\pm\theta}$-branes where μ is a mass of the adjoint chiral multiplet, the mass dimension of w is equal to 2. The mass dimension of y can be determined also by the boundary condition near $w = \infty$.

The precise relation between the holomorphic coordinates (v, y, w) and physical coordinates (4568910) are given by

$$v = \frac{x^4 + ix^5}{\ell_s^2}, \quad y = \mu^{2N_c} e^{\frac{x^6 - L_0 + ix^{10}}{2R}} \left(\frac{\sqrt{(x^4)^2 + (x^5)^2 + (x^6)^2} + x^6}{R} \right)^{\frac{N_f}{2}}, \quad w = \frac{x^8 + ix^9}{R\ell_s^2}$$

where R is a radius of eleventh direction and is given in terms of a string coupling and a string scale by $R = g_s \ell_s$ and we put R dependence here explicitly for M-theory description. As R goes to zero, one gets the type IIA brane description in previous section. Note that the presence of μ term in y provides the correct mass dimension above.

One of the complex structures of Taub-NUT space can be described by embedding it in a three complex dimensional space with coordinates (x, t, v). The M5-brane curve corresponding to the type IIA brane configuration shown in figure 1, in a background space [40] of

$$xt = (-1)^{N_f} v^4 \prod_{k=1}^{N_f} (v^2 - e_k^2) \tag{5.1}$$

where e_k is the position of the D6-branes in the v direction and the O6-plane $\mathbf{Z}_2$ symmetry action we discussed before can be rewritten as $(x, t, v) \to (t, x, -v)$,[19] can be characterized by [7]

$$t^3 + t^2 v^{N_c} + tv^2 (-1)^{N_c} v^{N_c} \prod_{k=1}^{N_f} (v - e_k) + v^6 \left[\prod_{k=1}^{N_f} (v - e_k) \right]^2 (-1)^{N_f} \prod_{k=1}^{N_f} (v + e_k) = 0. \tag{5.2}$$

Since the location of D4-branes in v direction is given by $v = 0$, the N_c-dependent terms in t^2 and t above have a simple form. For fixed x, the coordinate t corresponds to y and for fixed y, the coordinate x corresponds to $1/t$. Since the O6-plane plays the role of 4 D6-branes and D6-branes lift to the Taub-NUT space in the context of M-theory description, the power of v in the right hand side above (5.1) is equal to 4.[20] The above cubic equation (5.2) is a polynomial of degree 3 in t and this implies that there exist 3 solutions for t corresponding to three NS5-branes we are considering.[21]

[19] In a complex 3-dimensional space $\mathbf{C}^3$, the equation is characterized by $xt = \prod_{k=1}^{2N_f+4} (v - m_k)$ with $m_k = -e_k$. Now we are plugging the positions for the D6-branes including the contribution from the O6-plane. Then this can be written in terms of (5.1). Note that the position of O6-plane is located at $v = 0$.

[20] Of course, if we consider the negative charge for the O6-plane, this power will be -4.

[21] As observed in [7], the curve can be factorized under the particular circumstance for the coefficient

At $g_s \neq 0$, the NS5-branes can bend due to their interactions with the D4-branes and D6-branes. Let us consider the asymptotic behavior such that the rotated curve should have at $v \to \infty$ and $v = 0$. One can read off the behaviors of the left, middle, right NS5-branes respectively by considering the first two terms, the second and third terms, the last two terms from above cubic equation (5.2) respectively. Then the behavior of the supersymmetric M5-brane curves[22] can be summarized as follows:

1. $v \to \infty$ limit implies

$$w \to 0, \quad y \to -(-1)^{N_c} \Lambda_{N=1}^{2N_c-N_f-2} v^{N_f+2} + \cdots \quad \text{NS asympt. region.}$$

2. $w \to \infty$ limit implies

$$v \to m_f, \quad y \to -(-1)^{N_f-N_c} \Lambda_{N=1}^{4N_c-4N_f-8} w^{2N_f-N_c+4} + \cdots \quad NS'_L \text{ asympt. region,} \tag{5.3}$$
$$v \to -m_f, \; y \to -w^{N_c} + \cdots \quad NS'_R \text{ asympt. region.} \tag{5.4}$$

Here we inserted the appropriate scale $\Lambda_{N=1}$ in order to match the mass dimension above. Along the line of [5, 9], the left NS5'-brane is moved to the v direction holding everything else fixed, instead of moving D6-branes (and their mirrors). In this process, the corresponding mirrors are placed in appropriate way. We also shift the origin for x^6 direction and put the left D6-branes to the origin and denote the distance between the left D6-branes and NS5-brane as $L_0 + \Delta L$. That is, we denote the distance between the left D6-branes and NS5'-brane by L_0 and the distance between NS5'-brane and NS5-brane by ΔL, as in [5].

3. The map between the holomorphic and physical coordinates requires the condition [5]

$$y = 0 \qquad \text{only if} \qquad v = 0. \tag{5.5}$$

Since the nonsupersymmetric brane configuration in figure 3 implies that only the NS5'-branes are deformed by turning on the mass for the quarks, only NS5'-branes are nonholomorphic. The remaining NS5-brane and D6-branes remain unchanged. The ansatz for this nonholomorphic curve corresponding to two NS5'-branes where

functions appearing in the cubic equation:divisibility by $(t + t_0)$ where t_0 is v-independent. In this case, the quadratic piece in t corresponds to the Seiberg-Witten curve of $\mathcal{N} = 2$ SO(N_c) gauge theory with N_f flavors while the linear piece in t corresponds to the factorized middle NS5-brane. For the negative O6-plane charge, the gauge group will be changed into Sp(N_c) with N_f flavors. The M-theory descriptions in the presence of orientifold 4-plane [41, 42] are reproduced from the result of [35]. In this sense, the asymptotic behavior of M5-brane curve for the present gauge theory looks similar to those of symplectic or orthogonal gauge groups.

[22] Without O6-plane, the analysis for the two factor gauge groups [32] leads to the fact that the M5-brane curve [43] for $\mathcal{N} = 1$ SQCD with massive equal flavor masses can be obtained from the special limit of vanishing gauge coupling constant for the right factor, as expected. From the general result of [32], in particular, (4.39) and (4.40) of [32], the appropriate orientifold projection provides the corresponding the explicit relations of $v = v(w)$ and $y = y(w)$ for our gauge theory.

they have different boundary conditions characterized by (5.3) and (5.4) respectively can be made as follows [5]:

$$x^4 = f(s), \qquad x^5 = 0, \qquad x^8 + ix^9 = e^{i\frac{x^{10}}{2N_c R}} g(s), \qquad x^6 = s.$$

The unknown functions $f(s)$ and $g(s)$ can be determined by solving Euler-Lagrange equations for the action of surface parametrized by x^6 and x^{10} directions. The metric by the six dimensional transverse directions is given by the Taub-NUT space (45610) and flat two-dimensions (89).[23] The harmonic function appearing in the Taub-NUT space, sourced by the the left coincident N_f D6-branes located at $x^6 = 0$, O6-plane (which is equivalent to 4 D6-branes) located at $x^6 = \Delta L + L_0$, and the right coincident N_f D6-branes located at $x^6 = 2(\Delta L + L_0)$, can be written as explicitly, by putting the right charges and the locations of them,

$$V(s) = 1 + \frac{N_f R}{\sqrt{f(s)^2 + s^2}} + \frac{4R}{\sqrt{f(s)^2 + (s - \Delta L - L_0)^2}} + \frac{N_f R}{\sqrt{f(s)^2 + (s - 2\Delta L - 2L_0)^2}}. \tag{5.6}$$

Note that when we compare with the usual $\mathcal{N} = 1$ SQCD with massive flavors developed in [5], the last two terms are an extra piece coming from the effect of O6-plane and mirrors of D6-branes in our gauge theory.

When the left NS5-brane is located at $v = \frac{\Delta x}{\ell_s^2}$ from figure 3 and $f_L(s) = \Delta x$ which satisfies $f_L''(s) = 0$ because $f_L(s)$ doesn't depend on s, then the equation (A.3) of [5] which is valid for arbitrary form for $V(s)$ implies that $g_L'(s) = \frac{V}{2N_c R} g_L(s)$ with $V(s) = V(s)|_{f_L(s) = \Delta x}$. Then, this first order differential equation leads to the following solution for $g_L(s)$

$$\begin{aligned} g_L(s) \;=\; & R\ell_s^2 \left[(-1)^{-N_f + N_c - 1} \Lambda_{N=1}^{-4N_c + 4N_f + 8} \mu^{2N_c} e^{\frac{-L_0}{2R}} \left(\frac{1}{R} \right)^{\frac{N_f}{2}} \right]^{\frac{1}{2N_f - N_c + 4}} \\ & \times \left(s - l + \sqrt{(\Delta x)^2 + (s-l)^2} \right)^{\frac{2}{N_c}} \prod_{j=0}^{1} \left[s - 2lj + \sqrt{(\Delta x)^2 + (s - 2lj)^2} \right]^{\frac{N_f}{2N_c}} \end{aligned}$$

with $l \equiv \Delta L + L_0$. The s-independent integration constant is fixed by the boundary condition $y \to -(-1)^{N_f - N_c} \Lambda_{N=1}^{4N_c - 4N_f - 8} w^{2N_f - N_c + 4} + \cdots$ given by the classification 2 above (5.3). This is a simple solution $v = \frac{\Delta x}{\ell_s^2} = m_f$ and $y = -(-1)^{N_f - N_c} \Lambda_{N=1}^{4N_c - 4N_f - 8} w^{2N_f - N_c + 4}$. Even if Δx is equal to zero, the function $g_L(s)$ does not vanish. This implies w does not vanish and therefore y is not equal to zero. So this is, in fact, a contradiction with the above classification 3 in (5.5). In other words, the extra piece in the potential doesn't eliminate the instability from a new M5-brane mode occurring at some point during the continuation to M-theory description of SQCD.

[23] The metric by these six-dimensional transverse directions is given by $ds_6^2 = G_{AB} dX^A dX^B = V d\vec{r}^2 + V^{-1} \left(dx^{10} + \vec{\omega} \cdot d\vec{r} \right)^2 + (dx^8)^2 + (dx^9)^2$ where (456) directions are parametrized by $\vec{r}$ and the relation between the harmonic function V sourced by the D6-branes and the vector potential $\vec{\omega}$ is given by $\nabla \times \vec{\omega} = \nabla V$, as usual.

Furthermore, if the right NS5-brane is located at $v = -\frac{\Delta x}{\ell_s^2}$ and $f_R(s) = -\Delta x$ (note that this right NS5-brane is a mirror of the left NS5-brane) from figure 3, then it is easy to see that

$$g_R(s) = (-1)^{-\frac{1}{N_c}} R^{1-\frac{N_f}{2N_c}} \ell_s^2 \mu^2 e^{\frac{s-L_0}{2N_c R}} \times \left(s - l + \sqrt{(\Delta x)^2 + (s-l)^2}\right)^{\frac{2}{N_c}} \prod_{j=0}^{1} \left[s - 2lj + \sqrt{(\Delta x)^2 + (s-2lj)^2}\right]^{\frac{N_f}{2N_c}}$$

where l was defined before. In this case, we use the second property of the classification 2 given in (5.4) for the boundary condition. Since the potential form of $V(s)$ is given by the same expression, we are left with the same contradiction we met before.

Instead of imposing the boundary condition at large s, we require that M5-branes end on the D6-branes: $f(s)$ at $s = 0$ vanishes. For the case of $f_L(s) = cs$ which still satisfies $f_L''(s) = 0$ and $f_L'(s) \neq 0$, the (A.3) of [5] leads to the fact that there exists $g_L'(s) = \frac{\sqrt{1+f_L'(s)^2}}{2N_c R} V g_L(s)$ and when $L = L_0$ and $c = \frac{\Delta x}{L_0}$, this straight line solution will become the type IIA brane configuration in the $R \to 0$ limit. However, the behavior at infinity is different from the above classification 1 and 2. Similar analysis for the $f_R(s)$ where a straight line has an expression $f_R(s) = cs + d$ can be done. Therefore, the supersymmetric brane configuration and the nonsupersymmetric brane configuration are vacua of different theories because the boundary conditions at infinity are different.

In [5], they have tried to search for the possibility for the other solutions with the right boundary conditions by substituting the explicit form for the potential and have obtained third order nonlinear differential equation (A.5) of [5]. Surprisingly, the exact solution for the $f(s)$ with three parameters was found through (A.5)-(A.10) of [5]. However, since our potential has an extra piece in (5.6), compared with the one in [5], it is evident that the third order nonlinear differential equation for $f(s)$ cannot be solved exactly. This feature have occurred also in an example of [9]. The above solutions $f_{L,R}(s) = \pm\Delta x$, $f_L(s) = cs$ and $f_R(s) = cs + d$ are particular subfamily of the general solutions and, in principle, there could exist a solution having the correct boundary conditions both at infinity and at the D6-brane with $f_{L,R}''(s) \neq 0$. It seems to be difficult to construct this solution even if one uses the numerical analysis for the complicated diffferential equation corresponding to (A.5) of [5], as in [9].

6. Conclusions and outlook

We have constructed the type IIA brane configuration, presented in figure 3, corresponding to the meta-stable supersymmetry breaking vacua for $\mathcal{N} = 1$ SU(N_c) supersymmetric gauge theory with a symmetric flavor, a conjugate symmetric flavor and fundamental flavors. In doing this, the appropriate brane motion from the electric configuration to magnetic configuration has played the crucial role in order to obtain the right superpotential (4.3) that gives rise to the breaking of the supersymmetry. The O6-plane doesn't contribute to the creation of D4-branes because it is parallel to NS5'-brane so that the rank for the dual

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magnetic gauge group doesn't contain the constant term, contrary to the finite mass of adjoint field.

It is natural to ask whether the method for infinity limit of adjoint mass can apply to other supersymmetric gauge theories which can be realized in terms of type IIA string theory. As already observed in [4], for example, it is an open problem to construct the nonsupersymmetric brane configuration corresponding to the meta-stable supersymmetry breaking vacua for $\mathcal{N}=1$ SU(N_c) gauge theory with a conjugate symmetric flavor, an anti-symmetric flavor, fundamental flavors, and anti-fundamental flavors. This theory has a set of flat directions and the gauge group is broken to either SO(n) with an antisymmetric (adjoint) tensor and vectors or Sp(n) with a symmetric (adjoint) tensor and fundamentals [8]. We expect the meta-stable nonsupersymmetric vacua for these orthogonal or symplectic gauge theories should lead to the mesonic deformations [17, 29] of the $\mathcal{N}=1$ SU(N_c) SQCD with an adjoint field, by adding the appropriate orientifold 4-plane.

There exist many SQCD-like theories [8] where the Seiberg dual theories are known explicitly. It would be interesting to find out how to extract the simplified superpotential like as (4.3) by looking at its brane motion intuitively without dealing with various meson fields directly from gauge theory side.

Acknowledgments

We would like to A. Hanany, K. Landsteiner and D. Shih for discussions. This work was supported by grant No. R01-2006-000-10965-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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